SMOOTHING AND DECOMPOSITION OF DISTURBANCES OF THE INDICATOR DIAGRAMS WITH APPLICATION OF THE MOVING APPROXIMATING OBJECTS WITH BROKEN BONDS

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Abstract

There is defined a notion of a moving approximating object and there are also defined some types of bonds applied in construction the approximating objects in that work. There is presented the influence of using the broken bonds on the quality of smoothing the impulse disturbances and on the phase errors of approximation. The quality of the approximation, obtained by means of the Savitzky-Golay filters and by means of elaborated by Author approximating objects with the broken bonds, is compared on the examples of processing the indicator diagrams of the marine engines. It is proved that using the broken bonds, better smoothing of the impulse disturbances is obtained, without increasing the phase errors. That fact has a meaningful influence on the precision of determining the curve of intensity of the heat release. The presented examples show the possibility of application of the approximating objects with the broken bonds in multilevel decomposition of disturbances. It can be useful in processing the results of the pressure measurements on the indicator valves. It can also be used in processing the results of the stand tests charged with the disturbances which are not of the gauss type.

1. Moving multiple least squares approximation of the measurement series

The goal of the moving approximation is to determine the approximating value in a point belonging to the moving approximation interval of a fixed width [2, 4].

If we admit the minimum of sum of squares of deviations in the approximation criterion, then for p-th repetition of the approximation that criterion can be rewritten as follows:

$$MIN(S_{pj}) = MIN\left(\sum_{i=l_{pj}}^{i=r_{pj}} (\hat{y}_{(p-1)i} - \hat{y}_{pi})^2\right),$$
(1)

where:

p – the number of repetition of approximation (the number of passes), $p \ge 0$,

 $\hat{y}_{0i} = \tilde{y}_i$ – measured values,

 p_{ly} , r_{ap} – left and right end of the j – th approximation interval for the p-th repetition of approximation.

In general cases the approximation point is situated in the center of the approximation interval i.e. $l_{pj} = -k_p$, $r_{pj} = k_p$.

If the models of runs are unknown then they are often approximated by means of power or trigonometric polynomials. The moving mean value states the simplest example of the moving approximation. The moving mean value is a moving root-mean-square approximation by a power polynomials of the zero and first order if the approximation point lies inside the approximation interval. Savitzky and Golay [4] presented the algorithms of the moving approximation by power polynomials of higher order. They are reachable in lots of programs also in *Mathematica*.

2. Approximating objects based on the broken functions

The difficulties with approximation by the power polynomials or other functions of the interval of data with complex run can be overcome by modifying the approximating functions by imposing the bonds different from the equality of left- and right-sided values of derivatives in fixed points called knots. The bonds applied in spines are most known. In that case construction of bonds relies on introducing the discontinuity of the chosen derivatives in the knots which is equivalent with cutting those derivatives.

Sometimes the required approximating characteritic of the moving object can be obtained by using the riveting bonds [1, 3]. The riveting bond is defined as the condition of equality of values of a function in one or in a few points (knots). There are also exist the riveting derivatives.

The broken bond rely on giving a refracting index $w_j^{(m)} \neq 1$ for a chosen derivative of *m*-the order in a chosen knot *j*, which is equivalent with imposing a defined discontinuity of that derivative in the considering knot. The mathematical notation of that bond is as follows:

$$y_{j}^{(m)}(j) = w_{jm} y_{j-1}^{(m)}(j).$$
⁽²⁾

We assume that the values of a function in knots of the approximating object are continuous :

$$y_{j}^{(0)}(j) = y_{j-1}^{(0)}(j).$$
(3)

Figure 1. presents the scheme of the central approximating objects with the broken bonds.



Fig. 1. Scheme of the central approximating objects with the broken bonds

It is also assumed that the approximating object is central if the approximating point (the control point) is situated in the center of object interval. The approximating object is symmetrical if functions, knots, bonds are symmetric with respect to a center of the object. A segment of a broken line is the simplest example of a broken object. If we use the power polynomial of the third order to constructing the central approximating object with the broken knots then its recurrent equation is as follows:

$$y_{i} = a_{ji} + b_{ji}i + c_{ji}i^{2} + d_{ji}i^{3},$$

$$a_{j} = a_{j-snj} + q_{j}(1 - w_{j1})b_{j-snj} + q_{j}^{2}(1 - w_{j2} - 2(w_{j1} - w_{j2}))c_{j-1}q_{j}^{3}(1 - w_{j3} - 3((w_{j2} - w_{j3}) + (w_{j1} - w_{j3}) - 2(w_{j2} - w_{j3}))d_{j-snj},$$

$$b_{j} = w_{j1}b_{j-1} + 2q_{j}(w_{j1} - w_{j2})c_{j-1} + 3q_{j}^{2}(w_{j1} - w_{j3} - 2(w_{j2} - w_{j3})),$$

$$c_{j} = w_{j2}c_{j-1} + 3q_{j}(w_{j2} - w_{j3})d_{j-1},$$

$$d_{j} = w_{j3}d_{j-1}.$$
(4)

where:

j – the number of a knot,

 w_{j1} , w_{j2} , w_{j3} – bonds (constant) of the derivatives of 1, 2 and 3 order.

The analogous formulas can be easily written for the polynomials of different orders and also for trigonometric polynomials. The advantage of the power and trigonometric polynomials means existence of functions for positive and negative arguments.

To illustrate the influence of the polynomials order and broken knots on the smoothing results there are used the polynomials of first, third and fifth order with or without bonds.

All figures presents the runs (curves) defined by broken lines and the derivatives are differentials:

$$dy_{i} = \frac{y_{i} - y_{i-1}}{\alpha_{i} - \alpha_{i-1}}.$$
(5)

Analogous to dy_i there are determined the runs of $d2y_i$.

In that way there is omitted the influence of smoothing the runs of derivatives. In the elaborated algorithms the derivatives are determined from the runs smoothed by their interpolation or approximation by the power polynomials of fifth order with small number of degrees of freedom.

3. Applying the broken polynomials to determining the derivatives of the pressure runs

Smoothing the indicator diagrams is necessary when we want to determine the derivatives of first or higher orders. The problem of smoothing the indicator diagrams is specially connected with the problem of determining intensity of the heat release from the run of the cylinder pressure.

The measurements taken in exploitation conditions can be charged by different disturbances. The main source of the disturbances are: sensor with the measurement system, a/d converter, the measurement method, external disturbances. The disturbances implied from the used method are: disturbances from the gas passage, indicator valves. The external disturbances are: impulse disturbances from thyristor controllers, ignition systems of engines with spark ignition and inter-wire line crosstalks. The disturbances from the power supply network practically does not exist. The above mentioned kinds of disturbances are often during periodic or immediate measurements in exploitation conditions. The possibilities of eliminating them are limited even when their existence is discovered in the beginning of the measurement session. Trials of determining the derivatives from the pressure runs without smoothing are not successful. Figure 2. shows the comparison of the pressure runs and its derivatives before and after smoothing by means of the approximating object of [83B] type.

The disturbances of the pressure run p are not meaningful for evaluation the values pressure or for determining the value of the indicating pressure. They are meaningful for runs of the first order derivative. Then their existence states the barrier in determining the intensity of the heat release.

The pressure run p measured on the indicator valve is charged by disturbances from the pressure sensor, a/d converter, disturbances from the gas passage and from the indicator valve, impulse disturbances of unknown origin.

To smooth the p, there are used a few approximating objects and a few repetitions of approximation. In first two steps (Fig. 3) there are smoothed the disturbances of medium and high frequency.



Fig. 2. Comparison of the pressure runs and its derivatives before and after smoothing: p – the pressure measured on the indicator value of Sulzer engine 6A25/30 (n = 750 rpm, p_i = 1,8 MPa) $p_{[8B3]}$ – the run of the pressure smoothed by the approximating objects [8B3], dp, $dp_{[8B3]}$ – derivatives of first order for p and $p_{[8B3]}$



Fig. 3. The runs of dp for first two passes of approximation of p (data as in Fig. 1) with using chosen approximating objects (Table 1): $p_{[8B3]}$ – the run of pressure smoothed by [8B3], p_z – impulse disturbance

Characteristics of the applied approximating objects are presented in Table 1.

Lp.	Type of	Order of	Number	Number	Number	Number
	object	polynomial	of knots	of bonds	of passes	of points
						in object
1	[S0]	0	0	0	3	91
2	[S3]	3(2)	0	0	3	175
3	[S5]a	5(4)	0	0	3	175
4	[S5]b	5(4)	0	0	3	271
5	[2B4]	4	2	8	3	175
6	[8B3]	3	8	24	3	175

Table 1. Parameters of the approximating objects applied to approximation of the indicator diagram p (the engine data like for Fig. 2)

After the first step there are still large values of the derivative dp disturbances caused by the disturbances p_z . After the second step of approximation those disturbances are smoothed. Only the run $dp_{[S5]a}$ differs from them with strong oscillations.

To dampen them it is necessary to increase the object interval and after that increasing the value of k from 29 to 45 which smoothing those oscillations.

The comparable level of smoothing can be obtained without increasing the object interval, introducing the broken bonds to the object [S5] – the object [2B4]. Figure 3. shows only p [9B3] after approximation because for other objects the results of approximation are the same.

The level of smoothing the run is recognized as sufficient after the third step of approximation (Fig. 4).



Fig. 4. The runs of derivatives dp and d2p for the third pass of approximation of (data engine like for Fig. 1) with using chosen approximating objects (Table 1): p[8B3] – the run of pressure smoothed by means of the approximating objects [8B3]

As it is seen on Fig. 4. the oscillations of $dp_{[S5]a}$ do not decrease in comparison with the second step of approximation (Fig.3.). The highest level of smoothing is reached by the curve $dp_{[S0]}$, but in that case there are the largest phase distortions. It is needed to be noticed that in case of the moving mean value there are used smaller intervals in comparison with the other cases (Table. 1). The runs of dp for other approximating objects are practically the same. The differences in quality of their smoothing are seen after comparison the runs of d2p (Fig. 4). From Fig. 4. we conclude that the best dynamic characteristics are obtained for approximation by means of [8B3]. All obtained runs of d2p are still characterized by large oscillations and determining for example zero of that derivative with a sufficient accuracy is



Fig. 5. Comparison of the runs of intensity of the heat release dq and heat released Q for four approximating objects (Table 1) for engine data as from Fig. 1

thermodynamics treating the working factor as homogeneous perfect gas in a whole interval of lasting of the process. As we see from comparison of the runs only $dq_{[S0]}$ deviates in intervals of large dynamic changes of the pressure.



Fig. 6. Comparison of the runs of dq and Q obtained by using four approximating objects (table 2.) for the indicator diagram of the slow-speed vessel engine 6RTA76 (n =87,1 rpm, $p_i = 1,46$ MPa)

not possible. There are known proposals of using its first, starting from DMP, zero as the referred point of GMP.

The basic aim of smoothing the indicator diagrams is to obtain sufficiently smoothed runs of intensity of the heat release very useful in diagnostics of vessels engines. The runs of intensity of the heat release *dq* and the heat released obtained with using 0 four approximating objects (Table 1) for the considering run *p* are compared on Fig.5.

The runs of intensity of the heat release dq are determined on the basis of The first principle of

In the case of the released heat Q the concurrence of the obtained results is greater which follows from integrating the disturbances.

There are compared dq and Q determined from the indicator diagram of one of the cylinders of the slow-speed vessels engine using four chosen approximating objects on Fig. 6.

Choosing between obtaining the sufficient smoothness of dq and minimizing the phase and amplitude errors, the approximation by [6B3rp] was acknowledged as the best approximation (Table 2). The run $dq_{[S0]}$ is characterized by the greatest amplitude and phase errors.

Table 2. Parameters of the approximating objects applied to approximation of the indicator diagram p (the
engine data like for Fig. 6)

Lp.	Type of	Order of	Number	Number	Number	Number
	object	polynomial	of knots	of bonds	of passes	of points
						in object
1	[S0]	0	0	0	3	135
2	[S3]	3(2)	0	0	4	253
3	[S5]	5(4)	0	0	4	253
4	[6B3]rp	5(4)	6	12	4	253

The oscillation seen in the interval of 280-300 ca were not possible to smooth. It is the start of exhaust gases connected with a flow (a different form of energy balance) but that part can be extremely useful in diagnostics of supercharging and scavenging systems. Similarly to the previous case (Fig. 5.) the runs of Q are practically the same. Illustrating of those runs in larger scale will reveal a meaningful difference between $Q_{[S0]}$ and other runs of Q in the interval of accretion.

To avoid contradictions of approximation quality requirements various criteria can be used depending on aims and the interval of analysis of p.

4. Decomposition of disturbances

Obtaining the credible results of processing the data requires analysis of influence of disturbances on the results which is specially important for determining the derivatives. But in diagnostics the disturbances are the main aim.

The disturbances are determined as residuals between two chosen steps of approximation i.e. p = t and p = j

where:
$$t > j \ge 0$$
.

On the figure the are illustrated the disturbances determined as a result of approximation of p from Fig.1. by means of o [8B3].

 $Dy_{it} = y_i - y_t$,



Fig. 7. Runs of disturbances Dp determined as a result of approximation of p for data as for Fig.1. by means of the approximating object [8B3]

In that case the disturbances were divided into two kinds: high frequency disturbances Dp_{01} introducing by the measurement system, a/d converter and impulse disturbances of unknown origin. Disturbances Dp_{13} are introduced mainly by the gas passages and the indicator valve. They may also contain error of non adequate approximating object. Disturbances Dp_{01} are obtained after the first step of approximation while disturbances Dp_{13} after the third step.

(6)

For the second of the analyzed runs (Fig. 8) the disturbances are



determined after each of four steps of approximation.

Fig. 8. Runs of disturbances Dp determined as a result of approximation of p (for engine data as for Fig.6.) by means of the approximating object [8B3]

Disturbances Dp_{34} and Dp_{23} are analogous and can be presented as a summary disturbances Dp_{24} . However the loss of the useful signal caused by smoothing p by means of non adequate model signal has the main influence. Similar evidences are met in wavelet analysis of signals and they are often interpreted in a wrong way as a diagnostics evidences without explaining their physical properties [5].

The reason of difficulties with separation of disturbances lies in disturbances from other measurement systems (crosstalks). It is well seen on the runs of derivatives (Fig.9).



Fig. 9. Runs of dp and p for 4 steps of approximation of the indicator diagram by [8B3rp] (data of engine as on Fig. 6.), dp_z – disturbances caused by cross talks of signals in wire lines

As we see on Fig. 9. the comparisons of p, $p_1 - p_5$, disturbances of that type not can be observed directly on the measured run and appear on derivatives and runs of disturbances.

The approximation of moving enables divisions of disturbances onto a large number of levels, which is connected with the width of approximation interval the and with the number of repetitions and changes of parameters of approximation objects and also the types of objects used in next approximation steps.

5. Conclusions

In situation when the form of the mathematical model representing by the measured data is unknown or only partially known like in the case of the indicator diagram, for smoothing of the run it is suitable to use the moving approximation interval.

One of the possibilities of shaping the features of the approximating objects is introducing the broken bonds.

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